Neutrally buoyant particle in the boundary layer at a plate. I. Viscous interaction

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Abstract: The disturbance field created by a neutrally buoyant sphere, suspended in a boundary layer past a plate is investigated. The analysis shows that for particles with radii $R_p < (L/\mathrm{Re}_i^{5/4})$ (Re_i is the total Reynolds number of the background flow-stream), the disturbance is dominated by the viscous transverse interactions. Longitudinal viscous and convective terms appear in higher approximations with regard to dimensionless complexes, comprising the particle's radius, the boundary-layer thickness, and the length of the plate. These effects are just corrections to the basic viscous disturbance field.

Key words: Neutral buoyancy – boundary layer – viscous disturbance field

1. Introduction

The motion of a neutrally buoyant particle suspended in shearing flow has often been theoretically investigated [1–5]. These studies are connected with the choice of simple creeping background hydrodynamic fields (i.e., Poiseuille and Couette flows) and stem from the experimental observations of Segre and Silberberg [6]. In spite of the obvious fact that in many industrial processes and model experiments the hydrodynamics is far from being just creeping, the case of particles suspended in more complex flow, however, has not been attempted. One particularly interesting example in this sense, is the boundary-layer outer flow. There are some attempts to model particle's migration in incompressible laminar boundary-layer flow along a plate [7]. The analysis, however, concerns only heavy particles and neglects their dimensions.

The present paper proposes a consistent general approach to the interaction of neutrally buoyant solid particles suspended in boundary-layer flows past solid plates, starting from the complete fluid mechanics equations.

The empty boundary-layer flow past a plate is usually modeled via the Blasius expressions [7]. If a solid particle with finite dimensions is entrained in it, the background flow is perturbed. We are interested in neutrally buoyant particles, i.e., solids which are not sources of their own hydrodynamic fields in a stagnant fluid. Consequently, the single reason for the disturbance is the deformation of the external flow due to the finite dimensions of the particle. This disturbance results in a deflection of the particle's trajectory from the respective unperturbed streamline of the boundary-layer flow. Hence, the particle may either move towards the plate and remain entrapped in the boundary layer region, or it may be ejected out of it.

The present analysis outlines the parameters which determine the basic equations governing the particle's motion relative to the background flow. The case of prevailing viscous character of the interactions is studied. A procedure for the effective mathematical description of the disturbance, caused by a solid sphere entrained in a boundary layer past a plate is developed. A series of asymptotic equations is obtained which model the behavior of particles with different radii.

2. Statement of the problem

A solid sphere of radius R_p is freely suspended in a boundary layer formed by a stream with

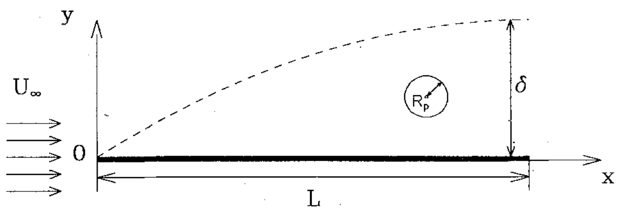

Fig. 1. Sketch of the problem. U_∞ – the background velocity; L – length of the plate; δ – boundary-layer thickness; R_p – radius of the solid sphere

velocity U_∞ past a plate of length L (see Fig. 1). The particle is small enough to be entrained in the boundary layer:

$$R_p \ll \delta, \quad \delta \sim (L/\sqrt{\mathrm{Re_i}}), \quad \mathrm{Re_i} = (U_\infty L/\nu) \,. \tag{1}$$

Here, δ is the boundary-layer thickness and $\mathrm{Re_i}$ is the integral Reynolds number of the undisturbed fluid flow. The assumption for the neutral buoyancy implies that $(\Delta\rho/\rho) < \mathrm{Fr}^2$ ($\Delta\rho$-density difference of the particle and the fluid, $\mathrm{Fr} = (U_\infty^2/Lg)^{1/2}$ is the Froude number for the background flow field). So far as the particle does not pertain its own velocity or rotation fields, the perturbation of the external fluid flow is caused only by the geometric fact that its finite dimensions obstruct the free propagation of the boundary-layer flow in its vicinity.

Let us assume that the disturbance of the basic stream flow by the solid sphere is weak and that the total velocity and pressure fields $(\tilde{u}, \tilde{v}, \tilde{p})$ may be presented as:

$$\tilde{u} = \bar{u} + u' + u, \; \tilde{v} = \bar{v} + v' + v,$$
$$\tilde{p} = \bar{p} + p' + p \,. \tag{2}$$

The following notations are used: $\bar{u}, \bar{v}, \bar{p}$ stand for the potential flow outside the boundary layer region; u', v', p' are the velocity components and the pressure inside the boundary layer and, according to Blasius [8], they have the form:

$$u' = U_\infty f'(\eta) \,,$$
$$v' = \frac{1}{2}\left(\frac{\nu U_\infty}{x}\right)^{1/2} (\eta f'(\eta) - f(\eta)) \,, \tag{3}$$

with ν being the kinematic viscosity of the fluid, and

$$\eta = y(U_\infty/\nu x)^{1/2} \tag{4}$$

is a dimensionless variable where the function $f(\eta)$ may be approximated with a power series of η: $f(\eta) = a\eta^4 + b\eta^3 + c\eta^2 + d\eta + e, a, b, c, d, e$ being constants; (u, v, p) is the disturbance field, due to the presence of the particle.

Upon introducing the expressions (2) in the Navier–Stokes equations [9] a system of differential equations for the disturbance field is obtained:

$$(\bar{u} + u')\frac{\partial u}{\partial x} + \frac{\partial u'}{\partial x}u + v'\frac{\partial u}{\partial y} + \frac{\partial u'}{\partial y}v + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{5a}$$

$$(\bar{u} + u')\frac{\partial v}{\partial x} + \frac{\partial v'}{\partial x}u + v'\frac{\partial v}{\partial y} + \frac{\partial v'}{\partial y}v + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$
$$= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{5b}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \,. \tag{5c}$$

The Blasius solution (Eqs. (3)–(4)) leads to the following expression for the local boundary-layer thickness [7]:

$$\delta(x) = \mathrm{const}\sqrt{\frac{\nu x}{U_\infty}} \,. \tag{12}$$

This formula and the assumption (1) give the limits of the region inside the boundary layer where the present study is adequate:

$$R_p \varepsilon \frac{\mathrm{Re_i^{1/2}}}{\mathrm{const}} \le x \le \frac{\mathrm{L}}{\mathrm{const}} \mathrm{Re_i} \,. \tag{13}$$

Here, x is measured along the length of the plate. Hence, the leading edge and the far downstream portions are not included in the present study.

3. Scaling parameters

The system (5) is analyzed using the methods of dimension and similarity theory [10]. Thus, the possible asymptotic cases are outlined, which may be treated analytically and have practical importance. The basic step in this direction is to choose the adequate scaling parameters for the quantities included in the general expressions (5) in order to render them dimensionless. The choice is based on a preliminary qualitative analysis of the system.

The model situation comprises a background hydrodynamic flow field (the boundary layer) and a small solid sphere of finite radius R_p. The assumption for the neutral buoyancy implies that the particle is not a source of its own hydrodynamic field and the result is a pure deformation disturbance of the outer boundary-layer flow field. An interaction of similar kind, but in a creeping outer flow, was studied by Faxen [9, 11]. He has found that in such circumstances an additional delay of the particle with regard to the velocity of the outer stream appears. Following this approach, we introduce the so-called Faxen's deformation velocity, which may be presented as:

$$v^f = \frac{R_p^2}{\mu}(\nabla p^0)_c = R_p^2(\Delta v^0)_c \,. \qquad (6)$$

The index "0" denotes that the respective quantity is calculated for the external flow field and "c" shows that it is taken at the point where the center of the sphere is situated.

For smaller particles, the disturbance field exhibits mainly viscous character. Therefore, it is particularly suitable to choose the components of the Faxen's deformation velocity (6) as scaling parameters for rendering the disturbance velocities dimensionless.

The background flow is given by the Blasius solution and therefore the dimensionless quantities (denoted with italics) acquire the following form:

$$u = \frac{u}{U_\infty \varepsilon^2(1+\Delta^2)}, \quad v = \frac{v}{U_\infty \varepsilon^2 \Delta(1+\Delta^2)} \qquad (7)$$

In these relations, we have $\Delta = (\delta/L)$, $\varepsilon = (R_p/\delta)$.

A characteristic feature of the concrete problem is the existence of two small parameters: ε stands for the relative dimension of the particle with regard to the boundary-layer thickness; Δ is the dimensionless thickness of the boundary layer.

The assumption for the relatively weak deformation perturbation of the basic flow implies that the disturbance includes a region in the vicinity of the solid particle, whose dimensions are scaled with R_p:

$$\Delta x = \Delta x \frac{R_p}{\Delta}, \quad \Delta y = \Delta y\, R_p \,. \qquad (8)$$

Generally, for any particle which lags behind or moves ahead of the basic stream, an additional interaction with the background flow appears. This is often referred to as lateral migration [12]. The nature of this phenomenon lies in the combined action of the viscous and convective terms in the equations of motion. In our case, the respective migration velocities are expressed as:

$$v_i^s = v_j \left(\frac{R_p^2}{\nu} \left| \frac{\partial v_j^0}{\partial x_i} \right| \right)^{1/2}, \qquad (9)$$

where v_i^s is the component of the Saffman's migration velocity, v_j^0 refers to the basic stream flow and v_j is the lag velocity of the particle (the difference of the particle's velocity and that of the external flow at the point where the particle center is situated), i, j are x, y-coordinates. Here, the lagging is due to the deformation interaction and $v_j = v_j^f$. The basic stream is the boundary layer flow past a plate and $v_j^0 = v_j'$. Because of the scaling relation (7) and the expressions (3)–(4), the respective velocity components in (9) acquire the form

$$u^s = u^s U_\infty \varepsilon^3 \Delta^{3/2}, \quad v^s = v^s U_\infty \varepsilon^3/\Delta^{1/2} \,. \qquad (10)$$

In summary, the interaction of a neutrally buoyant particle with the background flow field results in Faxen's deformation lagging behind the basic stream flow. In itself, this lagging leads to the appearance of Saffman's lateral migration in a direction perpendicular to the direction of the lag velocity. Hence, two basic possibilities for the choice of the disturbance velocity scaling parameters exist: Faxen's deformation velocity and Saffman's lateral migration rate. In this paper only the first possibility is discussed.

Thus, in what follows, we assume that the changes in the velocity components in the disturbance equations (5) are scaled with Faxen's deformation lagging of the particle in the boundary layer. This restricts the range of spheres' dimensions, for which the analysis is adequate. Because of the relation $v_i^f > v_i^s$, besides (1), an additional restriction of the dimensions of the particles is needed (see Fig. 2):

$$(R_p/L) < (1/\mathrm{Re}_i^{5/4}) \,. \qquad (11)$$

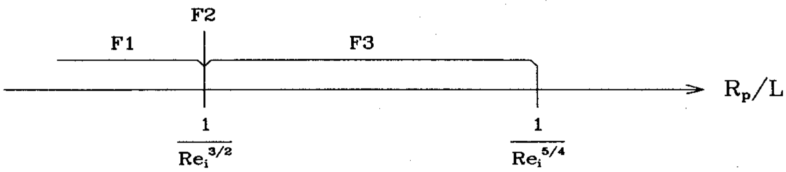

Fig. 2. Ranges of the particle dimensions. F1 – $R_p/L < 1/\mathrm{Re}_i^{3/2}$; F2 – $R_p/L \approx 1/\mathrm{Re}_i^{3/2}$; F3 – $1/\mathrm{Re}_i^{3/2} < R_p/L < 1/\mathrm{Re}_i^{5/4}$; Re_i – total Reynolds number of the background flow; L – length of the plate; R_p – radius of the solid sphere

The second case of prevailing Saffman's migrational scaling concerns the range of dimension:

$$(1/\mathrm{Re}_i^{5/4}) < (R_p/\mathrm{L}) < (1/\mathrm{Re}_i^{1/2}) . \tag{14}$$

It results in different sets of basic equations with a number of peculiarities and will be investigated in a separate paper (Part II, this volume).

4. Scaling analysis

Upon introducing the expressions (7) and (8) in Eq. (5), the system determining the disturbance field acquires the form:

$$\mathrm{Re}_p\left\{(\bar{u}+u')\frac{\partial u}{\partial x}+v'\frac{\partial u}{\partial y}+\varepsilon\left(\frac{\partial u'}{\partial x}u+\frac{\partial u'}{\partial y}v\right)+\varepsilon^2(1+\Delta^2)\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right\}$$
$$=-(1+\Delta^2+\mathrm{Re}_p(1+\varepsilon+\varepsilon^2+\Delta^2\varepsilon^2))\frac{\partial p}{\partial x}+\Delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} \tag{15a}$$

$$\mathrm{Re}_p\left\{(\bar{u}+u')\frac{\partial v}{\partial x}+v'\frac{\partial v}{\partial y}+\varepsilon\left(\frac{\partial v'}{\partial x}u+\frac{\partial v'}{\partial y}v\right)+\varepsilon^2(1+\Delta^2)\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)\right\}$$
$$=-(1+\Delta^2+\mathrm{Re}_p(1+\varepsilon+\varepsilon^2+\Delta^2\varepsilon^2))\frac{\partial p}{\partial y}+\Delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2} \tag{15b}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 . \tag{15c}$$

The above equations contain the quantity:

$$\mathrm{Re}_p=\varepsilon\Delta^2\,\mathrm{Re}_i . \tag{16}$$

It may be interpreted as local Reynolds number for the particle in the boundary layer. The integral Reynolds number for the outer stream is

$$\mathrm{Re}_i=\frac{1}{\Delta^2},\quad \text{therefore } \mathrm{Re}_p=\varepsilon \ll 1 . \tag{17}$$

As was already mentioned, the problem has two mutually independent small parameters – ε and Δ. According to the usual procedure of the perturbation analysis [14], the following relations between them are possible:

$$\text{F1}\quad \Delta^2>\varepsilon,\quad \text{i.e.,}\quad \frac{R_p}{L}<\frac{1}{\mathrm{Re}_i^{3/2}} \tag{18a}$$

$$\text{F2}\quad \Delta^2\sim\varepsilon,\quad \text{i.e.,}\quad \frac{R_p}{L}<\frac{1}{\mathrm{Re}_i^{3/2}} \tag{18b}$$

$$\text{F3}\quad \Delta^2<\varepsilon,\quad \text{i.e.,}\quad \frac{1}{\mathrm{Re}_i^{3/2}}<\frac{R_p}{L}<\frac{1}{\mathrm{Re}_i^{5/4}} . \tag{18c}$$

The velocity components and the pressure may be presented in different power series of ε and Δ, which result in various systems of asymptotic equations:

F1: $\Delta^2>\varepsilon$

The solution of (15) is sought in the form of:

$$u=u_0+\Delta^2u_1+\varepsilon u_2+\cdots$$
$$v=v_0+\Delta^2v_1+\varepsilon v_2+\cdots$$
$$p=p_0+\Delta^2p_1+\varepsilon p_2+\cdots . \tag{19}$$

Introducing the expressions (19) in (15), we obtain the following system of asymptotic equations:

F1.1 – zero approximation

$$\frac{\partial p_0}{\partial x}=\frac{\partial^2 u_0}{\partial y^2},\quad \frac{\partial p_0}{\partial y}=\frac{\partial^2 v_0}{\partial y^2},\quad \frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}=0 ; \tag{20}$$

F1.2 – first approximation

$$\frac{\partial p_0}{\partial x} + \frac{\partial p_1}{\partial x} = \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_0}{\partial x^2}$$

$$\frac{\partial p_0}{\partial y} + \frac{\partial p_1}{\partial y} = \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_0}{\partial x^2}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \; ; \qquad (21)$$

F1.3 – second approximation

$$(\bar{u} + u')\frac{\partial u_0}{\partial x} + v'\frac{\partial u_0}{\partial y} = -\frac{\partial p_0}{\partial x} - \frac{\partial p_2}{\partial x} + \frac{\partial^2 u_2}{\partial y^2}$$

$$(\bar{u} + u')\frac{\partial v_0}{\partial x} + v'\frac{\partial v_0}{\partial y} = -\frac{\partial p_0}{\partial y} - \frac{\partial p_2}{\partial y} + \frac{\partial^2 v_2}{\partial y^2}$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \, . \qquad (22)$$

These expressions present an asymptotic model for the interaction particle/boundary-layer flow in the case of relatively small particles (18a). The major changes (zero approximation) in the respective disturbance quantities are perpendicular to the plate, i.e., in the transverse direction, along the boundary-layer thickness. The governing equations (20) show that the interaction is determined by viscous transverse effects. In the first approximation changes in the longitudinal direction appear, but the viscous character of these changes is preserved. In the second approximation the convective terms are observed.

The asymptotic model F1 represents an easily solvable mathematical problem. Upon simple transformation any of the obtained systems are brought into Poisson-type differential equations:

$$\Delta u_{(i)} = A_{(i)}, \quad i = 0, 1, 2 \cdots \, . \qquad (23)$$

$$A_0 = f(x), \quad A_1 = f(x, y, v_0),$$

$$A_2 = f(x, y, u_0, v_0) \qquad (24)$$

F2: $\Delta^2 \sim \varepsilon$.

The respective quantities are expanded as power series of ε.

$$u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots$$

$$v = v_0 + \varepsilon v_1 + \varepsilon^2 v_2 + \cdots$$

$$p = p_0 + \varepsilon p_1 + \varepsilon^2 p_2 + \cdots \, . \qquad (25)$$

Hence, introducing (25) in (15), we obtain:

F2.1 – zero approximation

$$\frac{\partial p_0}{\partial x} = \frac{\partial^2 u_0}{\partial y^2}, \quad \frac{\partial p_0}{\partial y} = \frac{\partial^2 v_0}{\partial y^2}, \quad \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 \; ; \qquad (26)$$

F2.2 – first approximation

$$(\bar{u} + u')\frac{\partial u_0}{\partial x} + v'\frac{\partial u_0}{\partial y} = -\frac{\partial p_0}{\partial x} - \frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_0}{\partial x^2}$$

$$(\bar{u} + u')\frac{\partial v_0}{\partial x} + v'\frac{\partial v_0}{\partial y} = -\frac{\partial p_0}{\partial y} - \frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_0}{\partial x^2}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \; ; \qquad (27)$$

F2.3 – second approximation

$$(\bar{u} + u')\frac{\partial u_1}{\partial x} + v'\frac{\partial u_1}{\partial y} + \frac{\partial u'}{\partial x} u_0 + \frac{\partial u'}{\partial y} v_0 = -\frac{\partial p_0}{\partial x} - \frac{\partial p_1}{\partial x} - \frac{\partial p_2}{\partial x} + \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_1}{\partial x^2}$$

$$(\bar{u} + u')\frac{\partial v_1}{\partial x} + v'\frac{\partial v_1}{\partial y} + \frac{\partial v'}{\partial x} u_0 + \frac{\partial v'}{\partial y} v_0 = -\frac{\partial p_0}{\partial y} - \frac{\partial p_1}{\partial y} - \frac{\partial p_2}{\partial y} + \frac{\partial^2 v_2}{\partial y^2} + \frac{\partial^2 v_1}{\partial x^2}$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \, . \qquad (28)$$

This is an intermediate case. The leading order approximation (26) is the same as in F1. The first approximation, however, is quite different. The viscous longitudinal and transverse terms balance the convective terms. The same appears in the second approximation. Therefore, larger particles more seriously disturb the basic flow. The result is a more complex interaction, including both viscous and convective terms in the equations.

Upon transformation, the model F2 is also represented in a simple Poisson form:

$$\Delta u_{(i)} = B_{(i)}, \quad i = 0, 1, 2 \cdots \, . \qquad (29)$$

$$B_0 = f(x), \quad B_1 = f(x, y, u_0, v_0),$$

$$B_2 = f(x, y, u_0, v_0, u_1, v_1) \tag{30}$$

$F3$: $\Delta^2 < \varepsilon$.

The solution of (15) is sought in expansion series resembling Eqs. (19) (Case F1), but the places of Δ^2 and ε are interchanged:

$$u = u_0 + \varepsilon u_1 + \Delta^2 u_2 + \cdots$$

$$v = v_0 + \varepsilon v_1 + \Delta^2 v_2 + \cdots$$

$$p = p_0 + \varepsilon p_1 + \Delta^2 p_2 + \cdots . \tag{31}$$

Upon introducing (31) in (15), we obtain:

F3.1 – initial approximation

$$\frac{\partial p_0}{\partial x} = \frac{\partial^2 u_0}{\partial y^2}, \quad \frac{\partial p_0}{\partial y} = \frac{\partial^2 v_0}{\partial y^2}, \quad \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0 ; \tag{32}$$

F3.1 – first approximation

$$(\bar{u} + u')\frac{\partial u_0}{\partial x} + v'\frac{\partial u_0}{\partial y} = -\frac{\partial p_0}{\partial x} - \frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial y^2}$$

$$(\bar{u} + u')\frac{\partial v_0}{\partial x} + v'\frac{\partial v_0}{\partial y} = -\frac{\partial p_0}{\partial y} - \frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial y^2}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 . \tag{33}$$

F3.3 – second approximation

$$\frac{\partial p_0}{\partial x} + \frac{\partial p_2}{\partial x} = \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_0}{\partial x^2}$$

$$\frac{\partial p_0}{\partial y} + \frac{\partial p_2}{\partial y} = \frac{\partial^2 v_2}{\partial y^2} + \frac{\partial^2 v_0}{\partial x^2}$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 . \tag{34}$$

This case concerns the largest particles, adequately treated within the initial assumptions of the present analysis. The zero and the first approximation preserve the form already familiar from F2. A peculiarity is observed in the first-order asymptotic equations. The longitudinal viscous terms are absent there and they appear just in the second approximation.

The mathematical transformation of Eqs. (32)–(34) leads to the already familiar form of the Poisson equation:

$$\Delta u_{(i)} = C_{(i)}, \quad i = 0, 1, 2 \cdots . \tag{35}$$

$$C_0 = f(x), \quad C_1 = f(x, y, u_0, v_0),$$

$$C_2 = f(x, y, v_0) \tag{36}$$

Conclusions

The study in the present paper deals with particles completely imbedded in the boundary-layer flow. The interactions of the particles with the leading edge and the far downstream regions are not modeled here (see Eq. (13)).

The asymptotic model for the viscous interaction of a neutrally buoyant particle with the boundary-layer flow past a plate exhibits the following characteristics:

1) Within the assumption of the Faxen-deformation type of the perturbation field the analysis concerns particles whose dimensions are in the range of (14). The leading effect is the changes of the disturbance-velocity components in the transverse direction, perpendicular to the plate. Therefore, a serious transverse migration of the particle is to be expected. So far as the governing equations are regarded, the interaction has a clearly expressed viscous character for all dimensions of the particles.

2) In the sequential aproximations a clear differentiation of the particle's interaction type according to their dimensions is observed. For smaller particles (18a), the longitudinal viscous effects are more important and appear in the first approximation, while for larger particles (18c) they appear in the higher approximation. Convective terms balance the viscous members for larger particles earlier (in the first approximation) than for the smaller particles (in the second approximation).

3) Knowing the general characteristics of the background flow (U_∞, Re_i, L) and the particle's radius (R_p), one may immediately decide whether its disturbance-flow field is modeled via one of the proposed systems of equations. We may also choose the most suitable type of asymptotic behavior which describes adequately the hydrodynamic interaction.

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